Parameter-Insensitive Technique for Aircraft Sensor Fault Analysis

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A new method of analyzing faults in the m measurements of an nth-order system is presented. The proposed approach uses the estimation error space of each observer in a bank of observers to detect and isolate sensor faults. The designs are applied to a nonlinear model of an unmanned aircraft that has been described in previous publications. The reconfigurability of the aircraft sensor system is demonstrated, and the results show rapid recovery from a faulty sensor. The use of the observation error eliminates the need for state-space computations, thus producing an effective real-time fault monitor for fly-by-wire aircraft.

Introduction

IN an earlier paper,[1] a comparison of two techniques of instrument fault diagnosis (IFD) was made. This work is an extension of Patton and Willcox's idea of using analytical redundancy to design a match between m components of the observation error space instead of using state estimates directly as discussed by Clark,[3,4] Clark and Setzer,[5] Frank and Keller,[6] and Watanabe and Himmelblau.[7]

IFD in dynamic systems has received a significant amount of attention recently.[2-11] Most methods described in the literature discuss the analytical redundancy approach in preference to the use of redundant hardware. Analytical redundancy provides redundant (estimate) information from different measurements of a process, usually with observer or Kalman filter schemes. The commonly discussed state estimate solution to IFD is based on the principle of generating estimates of part or all of the system state vector from subsets of the measurements, which when compared with similar estimates from other observers can be used to monitor the health of an instrument. The problem with the state estimate solution to IFD arises as the observer requires a good linear model of the process, and it must also be assumed that the disturbances on the system are well modeled or else have an insignificant effect on plant parameter variations. These limitations cause the state estimate approach to be inadequate for many real engineering applications. Sensitivity to input-induced parameter variations causes uncertain errors between redundant state estimate vectors, and in an IFD scheme these errors could cause false signaling of an instrument fault. It becomes clear that the bandwidth of uncertain signals should be estimated prior to the IFD system design. The use of frequency domain sensitivity information in this way enables a robust approach to the observer design to be made. The conjecture used is that the "innovations" or prediction error signals contain all the information concerning the parameter variations of the process being identified and controlled. Attention is thus turned toward the use of an innovations-based approach to system fault diagnosis that has wide potential applications. By using a weighting of the measurement estimation error as a parity signal, sensor faults can be detected and located without the use of the whole state estimate vector. The observers designed in this way become fault monitor filters based on the estimation error. The use of a parity space approach has been published by other investigators.[9-11] However, the approach adopted in this work is based on "output zeroing," and a consideration of frequency domain parameter sensitivity is thought to be a new contribution. It is particularly important to note that the so-called "zero-sensitivity" direction in the parity space[9-11] must be valid for a wide range of nonlinear failures. This can only be achieved by considering frequency domain sensitivity through the use of band-limited filter structures as discussed by Jones.[8]

An observer based on the use of the estimation error for fault detection thus allows the utilization of a robust design approach. It is also important to distinguish between "parameter sensitivity" and "robustness"[12]: These ideas are applied in the design of robust observers for the IFD problem, as it is shown that parameter sensitivity is minimized and robustness to parameter and disturbance perturbations is achieved. The sensitivity measure S in this study is the vector of parity signals arising from weighted pairs of observation error signals in a bank of observers.

Aircraft System Description

The model described is based on an unmanned aircraft, contains the full-force aerodynamic motions in all axes, and includes lift stall characteristics. The control systems are designed using multivariable theory.[13,14] The longitudinal-lateral coupling has been retained in the simulation together with the full aerodynamic nonlinearities and band-limited turbulence inputs to replicate a realistic system environment. The coupling terms are omitted from the design.

The nonlinear equations used in the simulation have the following state variables: U (forward velocity), v (side velocity), w (normal velocity), p (roll rate), q (pitch rate), r (yaw rate), Θ (pitch angle), Φ (bank angle), and Ψ (heading angle). The IFD design is based on the following variational equations using the longitudinal motion:

$$\begin{bmatrix} \dot{U} \\ \dot{w} \\ \dot{q} \\ \dot{\Theta} \end{bmatrix} = \begin{bmatrix} x_U & x_w & 0 & -g \\ z_U & z_w & z_q + U & 0 \\ m_U & m_w & m_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ w \\ q \\ \Theta \end{bmatrix} + \begin{bmatrix} x_\zeta & 1 \\ z_\zeta & z_e \\ m_\zeta & m_e \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta \\ X_e \end{bmatrix} \tag{1}$$

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Equation (1) includes the specific thrust $X_e = T/m$ as input together with the corresponding normalized coefficients z_e, m_e. Short-period aircraft motions are rapid compared with changes in the forward speed U. These motions are considered to act with constant speed. This decomposition leads to the following subsystem on R^3:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\Theta} \end{bmatrix} = \begin{bmatrix} z_w & z_q + U & 0 \\ m_w & m_q & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \Theta \end{bmatrix} + \begin{bmatrix} z_\zeta \\ m_\zeta \\ 0 \end{bmatrix} \zeta \tag{2}$$

The coefficients of Eq. (2) are, in general, functions of the forward speed U and ζ is the elevator control signal. The eigenvalues of Eq. (2) closely approximate a subset of the spectrum of Eq. (1)—a result of the separation between the short-period and phugoid modes. A representation for the phugoid subsystem is

$$\begin{bmatrix} \dot{U} \\ \dot{\Theta} \end{bmatrix} = \begin{bmatrix} x_U - \dfrac{x_w\left(z_U m_q - m_U(z_q + U)\right)}{\left(m_q z_w - m_w(z_q + U)\right)} & -g \\ \dfrac{-(z_U m_w - m_U z_w)}{\left(m_q z_w - m_w(z_q + U)\right)} & 0 \end{bmatrix} \times \begin{bmatrix} U \\ \Theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_e \tag{3}$$

Equations (2) and (3) together with Eq. (1) give an overlapping decomposition which when used in a bank of observers provides analytical redundancy. The elements of the $A(x, u)$ and $B(x, u)$ (Jacobian) matrices are given by

$$A(x, u) = \frac{\delta f(x, u)}{\delta x}; \qquad B = \frac{\delta f(x, u)}{\delta u} \tag{4}$$

By evaluating these partial derivatives throughout the flight envelope, the trajectory sensitivity to parameter variations can be generated. Each of the elements of A and B may be considered to comprise a steady-state component (A_0, B_0) and a small perturbation component $(\Delta A, \Delta B)$ about this nominal operation, i.e.,

$$A = A_0 + \Delta A; \qquad B = B_0 + \Delta B \tag{5}$$

Figure 1 shows in graphical form how the parameters of the longitudinal motion of the aircraft, described earlier, vary during a simple maneuver. In this case, the aircraft performs a change in height of 10 m. This height change causes a significant variation in forward velocity compared with the nominal cruising speed of the aircraft. It can be seen that this maneuver causes the elements of the Jacobian A and B matrices to vary considerably from their nominal operating values for static flight. The diagram clearly shows that the trajectory sensitivity of the longitudinal motion system is influenced by the maneuver in two distinct frequency bands, according to the phugoid and short-period modes. Any IFD design based on a fixed linearized aircraft model will have its performance degraded from the theoretical projection when implemented in the full nonlinear system because of these parameter variations, and may not function within safe operating limits. It is therefore helpful to consider an IFD design based on a "worst case" maneuver condition as previously described.

Disturbance and Fault Models

The evaluation of an IFD scheme requires that other effects, apart from the aircraft dynamics, be modeled realistically. For this case, atmospheric disturbances and the instrument faults themselves can have as great an effect on the performance of an IFD scheme as do the dynamics of the system. It is important to use realistic gust and fault models in the simulation studies. Many different expressions have been formulated to fit measured wind gust data from different atmospheric conditions. The model used in this simulation has the Dryden power spectral density function.[15]

In previous work, IFD schemes have usually been evaluated using a single type of fault in a very simple form, e.g., a zeroed output or a step function or bias. In this work, models of four types of common instrument faults have been used. Although it is impossible to reproduce all types of possible instrument malfunctions, these models contain elements that would be common in most conceivable failures. The types of faults considered in this simulation are 1) slow drift, 2) step offset, 3) scale factor, 4) dead band, and 5) noise.

Parameter Sensitivity Considerations

The aircraft parameters have large variations during flight maneuvers. When a linear observer is applied to this system, the state estimates do not track the actual state vector closely. It is helpful to gain some insight into the cause of this parameter sensitivity. Consider the following state equations:

System (subsystem of plant):

$$\dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u; \qquad y = C_0 x$$
$$y \in R^m; \qquad u \in R^r; \qquad x \in R^n \tag{6}$$

where m = number of outputs, r = number of inputs, and n = number of states.

Observer:

$$\dot{\tilde{x}} = D\tilde{x} + B_0 u + KC_0 x$$
$$D = (A_0 - KC_0) \tag{7}$$

This gives

$$\dot{e} = (A_0 - KC_0)e + \Delta A x + \Delta B u$$

with

$$e = x - \tilde{x} \tag{8}$$

i.e., the observer has two additional inputs, ΔAx and ΔBu, giving rise to the possibility of large estimation errors. In order to design a robust IFD scheme, it is necessary to consider the frequency and time domain characteristics of the additional input terms. The elements of the trajectory sensitivity parameters vary at different rates in response to maneuvers. In particular, the elements of $\Delta A(x, u)$ have relatively high frequency variations. These spectral characteristics allow the ΔAx, ΔBu terms to be separated into two distinct frequency bands such that the two disturbance terms are processed independently.

Analytical Redundancy Using Bank of Single-Input Observers

It is assumed that the four longitudinal motion states have independent measurements. An observer can be designed for each measurement (taken as a single input). The state vector $x \in R^4$ can be reconstructed from a number of measurements —in each case with differing degrees of observability. In the case of the perfectly matched system with linear observer, reasonable estimation results can be obtained.[6] However, in the nonlinear case, the estimation error e can vary markedly between observer systems due to parameter sensitivity.

Significant variations, in $B(x, u)$, for example, will affect the performance of a fault detection scheme. Difficulties arise as the estimation error dynamics are excited by mismatches in the (assumed) model parameters. It is thus difficult to place a low threshold on the state estimates of an observer to give a sensitive measure of an instrument fault. A simplified analysis of this system is given as follows. Consider two observers with state estimate vectors x_1 and $\tilde{x}_2$, respectively. The estimator dynamics are then given by

$$\dot{\tilde{x}}_1 = A_0\tilde{x}_1 + B_0 u + K_1(C_{01}x - C_{01}\tilde{x}_1)$$
$$\dot{\tilde{x}}_2 = A_0\tilde{x}_2 + B_0 u + K_2(C_{02}x - C_{02}\tilde{x}_2) \qquad (9)$$

Now define

$$e_1 = x - \tilde{x}_1;\ e_2 = x - \tilde{x}_2$$
$$e_{12} = e_1 - e_2 = \tilde{x}_2 - \tilde{x}_1$$

For a nonlinear process with small parameter deviations

$$\dot{e}_1 = (A_0 - K_1 C_{01})e_1 + \Delta A x + \Delta B u$$
$$\dot{e}_2 = (A_0 - K_2 C_{02})e_2 + \Delta A x + \Delta B u \qquad (10)$$

In the case of no measurement faults

$$\dot{e}_{12} = A_0 e_{12} - K_1 C_{01} e_1 + K_2 C_{02} e_2 \qquad (11)$$

Now e_1 and e_2 depend on the inputs ΔAx and ΔBu to different extents as $K_1C_{01} \neq K_2C_{02}$. Furthermore, it is not simple, in general, to arrange for $K_1C_{01}e_1$ to equal $K_2C_{02}e_2$. This approach based on differencing the state estimates of dissimilar observers is thus not well suited to the application of instrument fault analysis. Methods of overcoming these problems cannot be generalized and are very complex in nature, as discussed by Watanabe and Himmelblau.[7]

Analytical Redundancy in Observation Error Space

By using the state-space in an IFD system, redundant parameter-sensitive state estimate information is generated. It is natural to consider the use of redundant information in the estimation error generated by the observer. The system becomes one of selecting observer gains to give a modal match between the residual signals of a two-input observer. The s domain design provides the appropriate weighting W between the residuals e_1 and e_2 such that in the no-fault condition, the weighted difference signal becomes insensitive to process parameter deviations. The filter has a robustness property that is easily understood using the frequency domain. A compromise must be met between rapid tracking and sensitivity to system uncertainties. The decomposition of the longitudinal motion is used to design a bank of reduced-order observers, each with different input pairs.

The w, q subsystem observer is driven by the w, q measurements. A w, q, Θ subsystem observer is driven by the w, Θ measurements, and a w, q, Θ subsystem is driven by the q, Θ measurements. The U, Θ (phugoid) subsystem is driven by the U, Θ measurements. The resulting scheme is shown in Fig. 2. The combination of four correctly designed observer subsystems can be used to detect and isolate all faults in the sensor system being analyzed.

A fault flagged by the w, q subsystem will relate to a fault flagged in the w, q, Θ subsystem with w, Θ as inputs if the w sensor has failed. Similar reasoning can be applied to the unique location of sensor faults. The basis of the method can also be applied to component fault diagnosis using decentralized observers.

The design is based on the observer model given by Eqs. (6–8). It is assumed that the uncertain terms ΔAx and ΔBu account for weak coupling terms not included by forming the reduced-order subsystems of Eqs. (2) and (3), in addition to representing the effect of parameter variations.

The problem is one of assigning the eigenstructure of each observer in Fig. 2 such that a weighted combination of the innovations of each observer gives a "null signal" in the no-fault condition. This is a specific application of the "output zeroing"[16,17] problem applied to the estimation error space. An additional consideration is that the design is made insensitive to parameter variations and sensitive to measurement faults. There are two classes of design to consider: 1) subsystems for which the number of measurements m is equal to the number of states n, and 2) subsystems for which $n - m > 1$, i.e., for which there is at least first-order redundancy in the state estimation system.

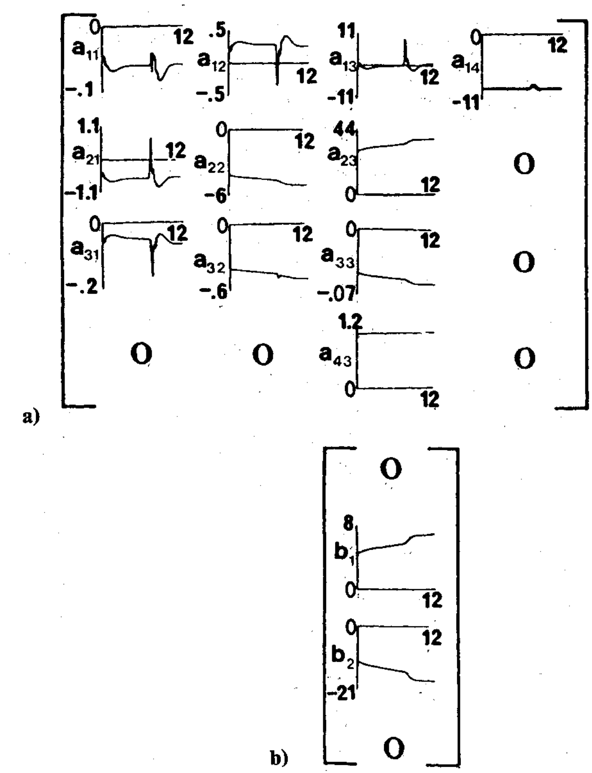

Fig. 1 System matrices: a) $A(t)$ and b) $B(t)$.

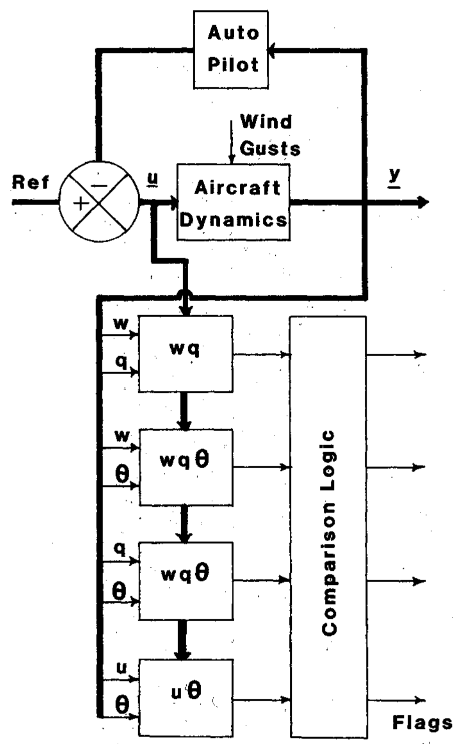

Fig. 2 Innovations-based IFD scheme.

In case 1 ($n = m$), there is freedom in assigning arbitrarily all n observer modes if (A_0, C_0) is an observable pair.[18,19] On the other hand, for case 2, the arbitrary (but unique) assignment is achieved by assigning *at least* $n - m$ eigenvectors and all n observable modes. The maximum number of assignable null-space eigenvectors possible is thus $n - 1$ for $m = 2$, $n - 2$ for $m = 3$, etc. We define the following to be a fault detection signal:

$$F(s) = WC_0E(s) = 0 \tag{12}$$

with
$$E(s) = T(sI_n - J)^{-1}T^{-1}B_0u^*(s) \tag{13}$$

where * denotes the convolution operation, and for convenience of notation, $B_0u^*(s) = B_0\Delta B^*u(s)$. T is the generalized eigenvector basis set that maps the original state-space into Jordan canonical form with system matrix J. In all cases, it is necessary to design the fault monitor such that the fault parity signal is insensitive to parameter variations. However, it must also be sensitive to sensor fault signals. This sensitivity is achieved by arranging the eigenvector corresponding to the redundant mode such that there is enough of this mode in the parity signal. The range space eigenvalue location determines the speed of the response to failures. However, its position and hence filter bandwidth also prescribe the degree of parameter sensitivity. We can now consider cases 1 and 2 with examples to illustrate the technique used.

Case 1: $n = m$

If the matrix D is chosen to be diagonal with distinct eigenvalues, then the matrix J becomes the spectral matrix Λ:

$$D = (A_0 - KC_0) = \Lambda \tag{14}$$

It then follows that

$$E(s) = (sI_n - \Lambda)^{-1}B_0u^*(s) \tag{15}$$

Thus, for $F(s) = WC_0E(s) = 0$, it also follows that for this special case it is necessary for W and C_0B_0 to satisfy

$$WC_0B_0 = 0 \tag{16}$$

The problem thus becomes that of choosing W to satisfy Eq. (12). This result holds for $n = m$, for (A_0, C_0) an observable pair and for $(C_0, B_0) = \text{rank }(B_0)$. For Eq. (16) to have a nontrivial solution set, it is necessary that $B_0 \in N(WC_0)$. Some of the dynamics of the estimation error system may or may not lie in this null space. An additional property can be achieved if $n - r$ eigenvectors are assigned to the null space of WC_0. It is important for parameter sensitivity considerations to reduce the order of the estimation error system (making r modes unobservable), reducing the sensitivity to parameter changes. An example serves to illustrate the design for this case. The results of applying this design to the nonlinear aircraft system are described in the next section.

Example 1: $n = m = 2$, $r = 1$

$$A_0 = \begin{bmatrix} z_w & z_q + U \\ m_w & m_q \end{bmatrix} = \begin{bmatrix} -2.930 & 32.770 \\ -0.416 & -0.645 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 5.318 \\ -13.580 \end{bmatrix}; \qquad C_0 = I_2 \tag{17}$$

The design steps are as follows:

1) Choose W such that $WC_0B_0 = 0$, e.g.,

$$W = [13.58 \quad 5.318] \tag{18}$$

2) Assign the eigenstructure of $D = (A_0 - KC_0)$ such that

$$M = \begin{bmatrix} 5.318 & 0 \\ -13.580 & 1 \end{bmatrix} \tag{19}$$

with

$$D = \Lambda = \begin{bmatrix} -30 & 0 \\ 0 & -2 \end{bmatrix} \tag{20}$$

This yields

$$K = \begin{bmatrix} 27.070 & 32.770 \\ -0.416 & 1.355 \end{bmatrix} \tag{21}$$

Finally, it can be seen from Eqs. (15) and (16) that

$$F(s) = WC_0E(s) = 0 \tag{22}$$

as

$$E(s) = \begin{bmatrix} e_1(s) \\ e_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{5.318(s+30)u^*(s)}{(s+30)(s+2)} \\[2ex] \dfrac{-13.58(s+30)u^*(s)}{(s+30)(s+2)} \end{bmatrix} \tag{23}$$

The design results in a limited bandwidth (robustness) filter, which minimizes the sensitivity of the fault signal $F(s)$ to parameter changes ΔBu and ΔAx. In the nominal (matched) case, for $\Delta A = \Delta B = 0$, the mode $s = -30$ is made unobservable in the estimation error, and since ΔBu has lower frequency variations than ΔAx, the effects of ΔAx can be considered negligible. The uncancelled mode $s = -2$ can be considered as a *redundant mode*, with which the match is created. When a sensor fault occurs, Eq. (8) becomes

$$e = (A_0 - KC_0)e + \Delta Ax + \Delta Bu + K\Delta Cx \tag{24}$$

Case 2: $n > m$

For this case, there is a reduction in eigenstructure assignment freedom compared with case 1.[18,19] The input matrix B_0 can now no longer be used to design a weighting matrix W. It is also necessary to exploit an additional result as follows:

$$E(s) = T(sI_n - J)^{-1}T^{-1}B_0u^*(s) \tag{25}$$

We can select a transformation matrix T such that WC_0 becomes a left annihilator of $E(s)$. It is feasible to assign a *generalized* eigenstructure[20] to D such that

$$WC_0m_i = 0 \tag{26}$$

where $i = 1, m - 1$, and m_i are distinct right eigenvectors of $D = (A_0 - KC_0)$. It then follows that

$$WC_0E(s) = 0 \tag{27}$$

Motions of the estimation error lying in the null space of WC_0 will be insensitive to parameter changes acting on A_0, provided that they also act in the range space of C_0. This is the dual of an important result that has been quoted previously for the control problem.[17] Motions close to the null space will also have a low sensitivity to parameter variations due to the narrow bandwidth assignment of the range space eigenvalue.

This is achieved as follows:

1) Consider v_i to be a *right* eigenvector of the dual (control) system given by

$$z = \left(A_0^T - C_0^TK^T\right)z = D^Tz, \qquad z \in R^n \tag{28}$$

It follows by transposition that v_i^T is a *left* eigenvector of the observer problem, i.e.,

$$v_i^T D = v_i^T \lambda_i \tag{29}$$

2) The aforementioned property leads to the well-known result that the dual sets of right eigenvectors of D and D^T corresponding to distinct eigenvalues are orthogonal. This result is extended to the case in which m eigenvectors form a simple degenerate set with multiplicity m. This is achieved using generalized eigenvectors of D. The orthogonality condition is

$$\langle m_i, v_k \rangle = 0, \qquad i \neq k$$

for

$$D^T v_i = v_i \lambda_i$$

$$D m_i = m_i \lambda_i \tag{30}$$

3) If v_i is an assignable *right* eigenvector of the dual system D^T, then *at least one* right eigenvector can be assigned to D^T such that $v_i m_k = 0$ and $i \neq k$. The eigenvectors of this null space are thus orthogonal to the equivalent range space in the dual problem. Thus, one eigenvector of D^T is assigned, and the generalized right eigenvectors of D cause m pole-zero cancellations in Eq. (27) such that the order of $F(s)$ is reduced from n to 1. An example serves to illustrate the procedure, and results based on this design are presented in this paper.

Example 2: $n = 3$, $m = 2$, $r = 1$

$$A_0 = \begin{bmatrix} -2.930 & 32.770 & 0 \\ -0.416 & -0.645 & 0 \\ 0.000 & 1.000 & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 5.318 \\ -13.580 \\ 0.000 \end{bmatrix} \qquad C_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{31}$$

In this example with $m = 2$ and $n = 3$, one eigenvector corresponding to $s = -20$ is assigned with multiplicity two. The range space of WC_0 is thus a first-order dynamic, as required. This arises because the assignment of two linearly dependent eigenvectors causes two pole-zero pairs to cancel, thus reducing the observability index of the observer to 1.

The two cancelling modes must be made faster than the remaining (range space) mode, and $s_1 = -20$, $s_2 = -20$, $s_3 = -2$ is a reasonable choice for this problem. The slow mode will allow the estimation error components $e_1(s)$ or $e_3(s)$ to respond to sensor faults while filtering the effect of rapid parameter variations. The eigenvector assignment is completed by assigning the three eigenvalues and one eigenvector v_3 corresponding to the $s = -2$ (slow) mode to the dual (control) problem as follows:

$$\text{for } s = -2, \; v_3^T = [-0.086 \quad 1 \quad 1.404] \tag{32}$$

which provides a strong weighting for the $s = -2$ mode, as required. In practice, the vector $v_3^T = [p_1 \quad 1 \quad p_2]$ is assigned. The elements p_1, p_2 must be nonzero; otherwise an arbitrary choice can be made as long as v_3 is an allowable eigenvector of D^T. The ratio p_1 to p_2 gives the weighting factor of the two measurement estimation error signals. The dual closed-loop system matrix is

$$D^T = \begin{bmatrix} -29.758 & -2.858 & 0.329 \\ 32.770 & -0.645 & 1.000 \\ -71.943 & 7.167 & -11.609 \end{bmatrix} \tag{33}$$

and the modal matrix is

$$V = \begin{matrix} (v_1) & (v_2) & (v_3) \\ \end{matrix}$$
$$V = \begin{bmatrix} 0.095 & 0.095 & -0.061 \\ -0.213 & -0.213 & 0.712 \\ 1.000 & 1.000 & 1.000 \end{bmatrix} \tag{34}$$
$$(s=-20) \qquad (s=-20) \qquad (s=-2)$$

The (transformed) observer problem has system matrix D. The generalized modal matrix is

$$\begin{matrix} (m_1) & (m_2) & (m_3) \end{matrix}$$
$$T = \begin{bmatrix} 1.000 & 1.000 & 1.000 \\ 0.169 & 0.377 & 1.196 \\ -0.059 & -0.208 & 0.160 \end{bmatrix} \tag{35}$$
$$(s=-20) \qquad\qquad (s=-2)$$

m_2 is a *generalized* eigenvector based on the mode $s = -20$ which, along with m_1, satisfies $(D - \lambda I)^2 m_i = 0$ for $i = 1,2$ and $\lambda = -20$. The vectors m_1, m_2, and m_3 form a basis for the Jordan canonical form for D[17], i.e.,

$$J = T^{-1} D T \tag{36}$$

The orthogonality condition of Eq. (30) is satisfied with

$$v_3^T m_1 = v_3^T m_2 = 0 \tag{37}$$

i.e., m_1 and $m_2 \in N(v_3^T)$. Furthermore, $WC_0 m_1 = 0$ is satisfied with

$$W = [1 \quad 16.94] \tag{38}$$

The parity signals are thus given by

$$C_0 E(s) = \begin{bmatrix} e_1(s) \\ e_3(s) \end{bmatrix} \approx \begin{bmatrix} \dfrac{-16.94 u^*(s)}{(s+2)} \\[2ex] \dfrac{u^*(s)}{(s+2)} \end{bmatrix} \tag{39}$$

Note that $F(s) = WC_0 E(s) \approx 0$, $\forall s$ corresponding to the nominal system. Once again, first-order analytical redundancy has been achieved using the range space mode of $s = -2$. For the general system with $n > m$, this form of solution can be obtained as long as (A_0, C_0) is an observable pair and *at least* $n - m$ right eigenvectors span $N(WC_0)$. This can be achieved using the assignment freedom conditions provided by Andry et al.[19] In order to achieve the required eigenvector multiplicity, generalized eigenvector assignment[20] must be used.

Robustness Considerations

The design of $F(s) = WC_0 E(s) = 0$ is known as the "output zeroing" problem, which has received considerable attention in the literature.[16,17] The property is well known, for example in the design of variable structure (sliding mode) (VSS) controllers. For the VSS problem, the output zeroing approach provides the design of the required range space and null space dynamics such that on reaching the hyperplanes of the null space, the trajectory will slide towards the origin. In VSS control, the effect of parameter variations is accounted for by switching the control law to force the state trajectory back onto the null-space manifold. This is necessary as, for the control problem, parameter variations lying outside $R(B)$ can pull the trajectory off the null-space hyperplane(s). This is particularly useful in the control problem as the system is designed with fast range space modes. By designing reduced-order subsystems and using these models in observers, the

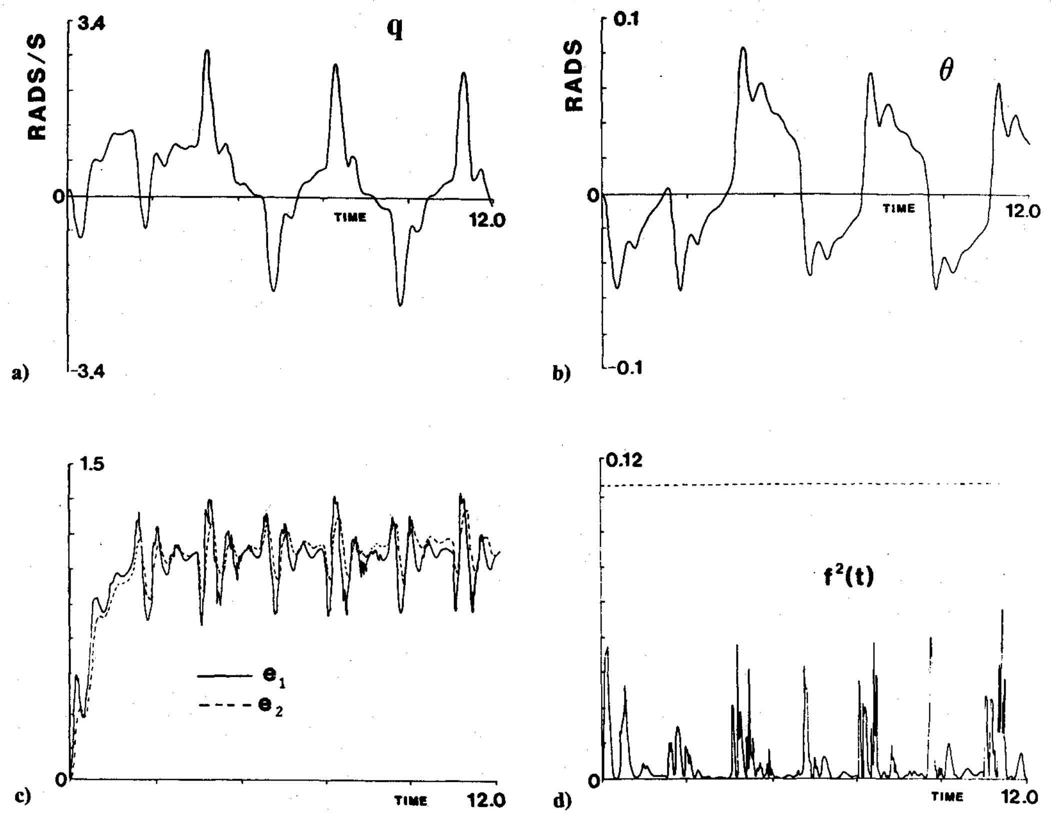

Fig. 3 No sensor fault: a) pitch rate, b) pitch angle, c) weighted estimation error elements, and d) squared fault detect signal.

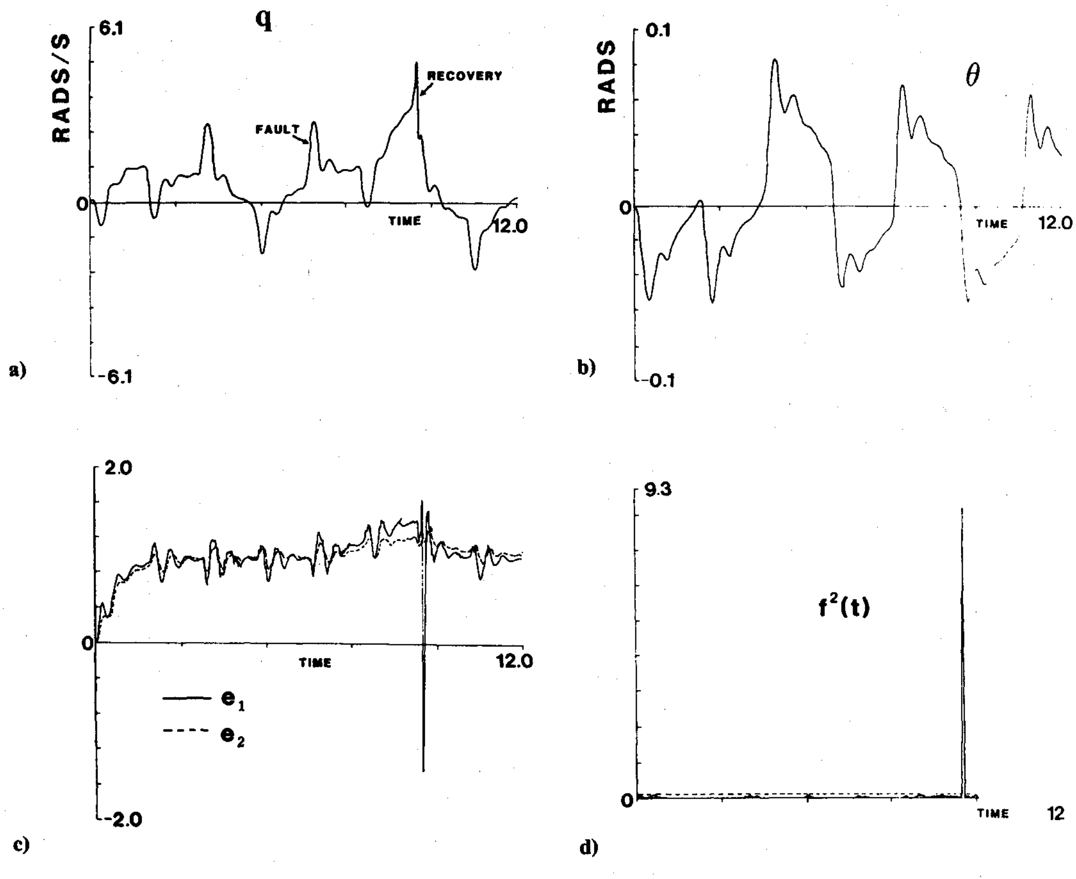

Fig. 4 Slow gyro drift fault: a) pitch rate, b) pitch angle, c) weighted estimation error elements, and d) squared fault detect signal.

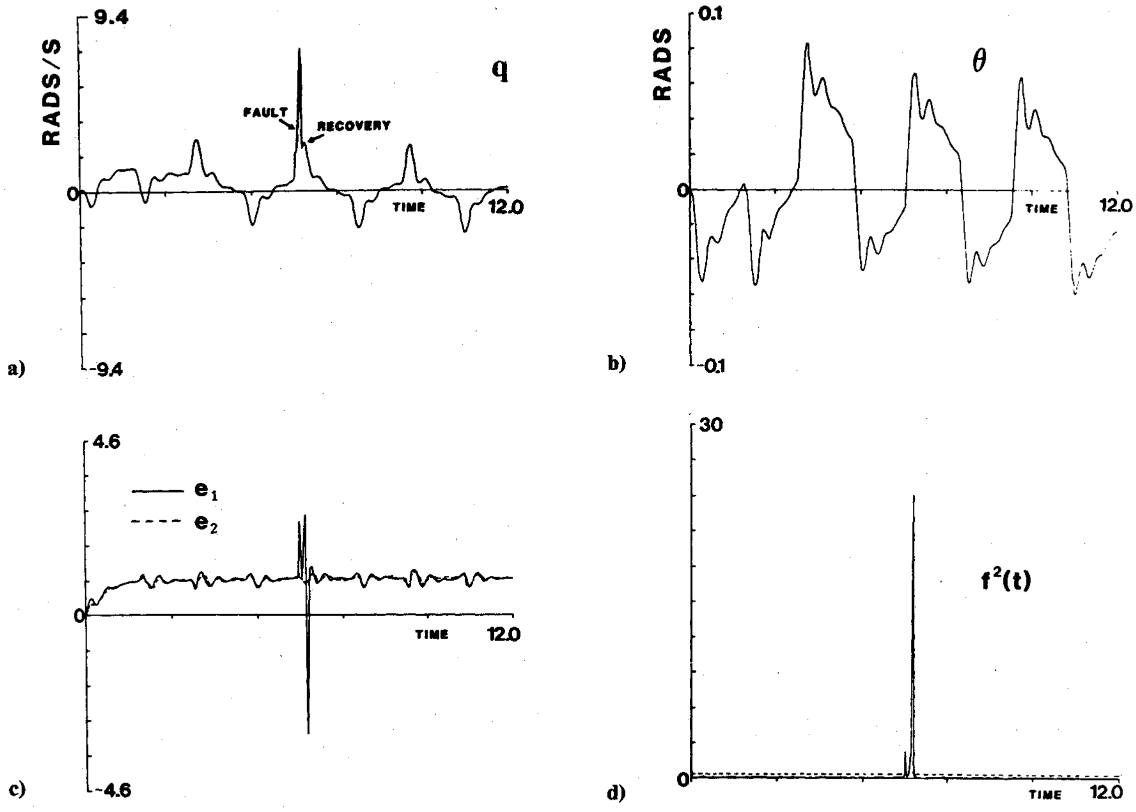

Fig. 5 Step offset gyro fault: a) pitch rate, b) pitch angle, c) weighted estimation error elements, and d) squared fault detect signal.

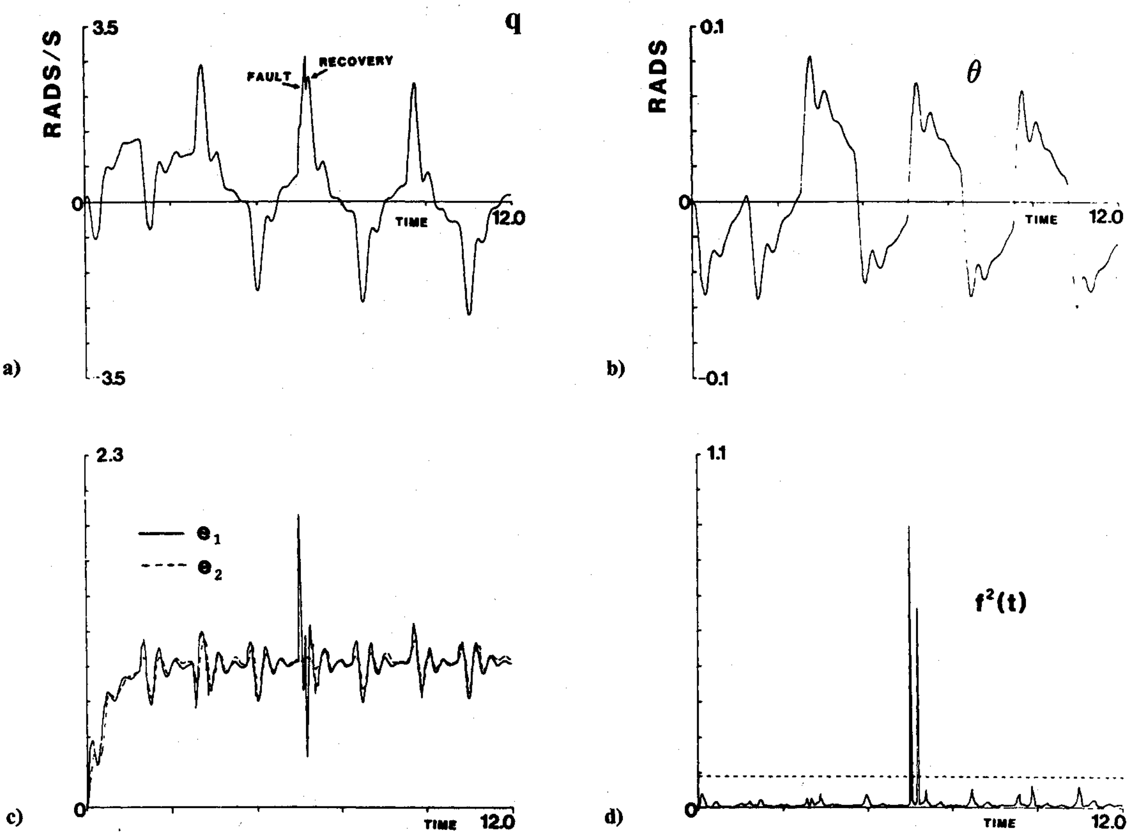

Fig. 6 Scale factor gyro fault: a) pitch rate, b) pitch angle, c) weighted estimation error elements, and d) squared fault detect signal.

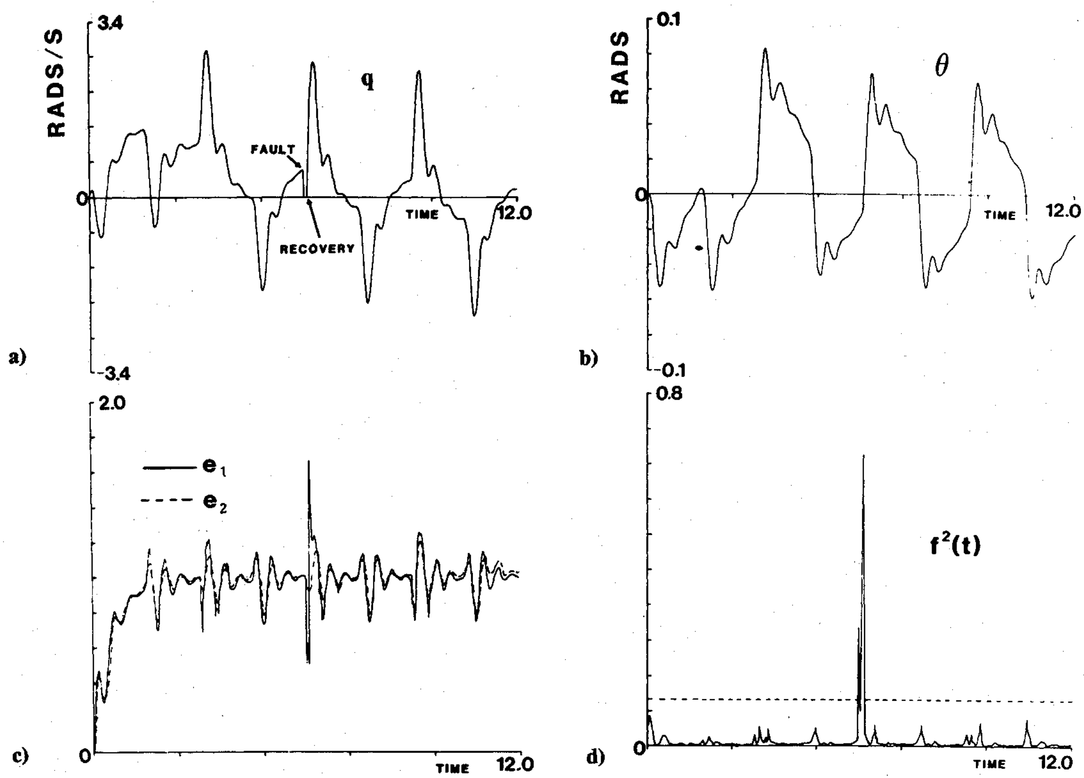

Fig. 7 Dead band gyro fault: a) pitch rate, b) pitch angle, c) weighted estimation error elements, and d) squared fault detect signal.

output zeroing approach can also be applied to the fault diagnosis problem. Switched feedback in the observer is undesirable as this would have the effect of desensitizing the estimation error system to sensor faults. By using first-order analytical redundancy in each observer, the assignment of the slow redundant pole can be made to correspond to a narrow band filter with low sensitivity to parameter variations. If the fault detection signal for the ith observer subsystem is $F_i(s)$, then the overall detection signal (covering the whole system measurement) can be defined as

$$S = [F_1(s), F_2(s), \ldots, F_k(s)]^T = 0 \qquad (40)$$

where k is the number of permutations of arranging the measurements taken m at a time. S is thus a sensitivity vector.

IFD System Performance

Figures 3–7 show the result obtained using case 2 and the new scheme previously described with the nonlinear aircraft model using a band-limited linear turbulence model and maneuver demand pilotic inputs. The turbulence signals are generated using white noise driving the Dryden spectrum filter with a 28 rad/s bandwidth. The pilotic inputs result from a 3 m height change demand after every two seconds. In each of the diagrams, "a" and "b" show the w and Θ measurements respectively, "c" shows each pair of measurement error vector elements, and "d" shows the square of the weighted errors (i.e., $[f(t)]^2$) making up the fault-detect signal. "d" also shows a fault threshold level which, if exceeded, signifies a fault condition on one of the sensors. Figures 4–7 correspond to the fault situation. The onset of each fault is shown on the time history of the faulty sensor, together with the time at which the fault is detected and a reliable redundant sensor switched into operation for recovery. In Fig. 3, the measurements and innovations signals for a no-fault situation are shown. It can be seen that the two weighted observation error (innovations) signals appear to be closely matched and that the fault-detect signal shown in "d" is very small, despite the large parameter variations and noise. In the linear system case (not shown), the weighted innovations signals match exactly in the no-fault case. The degree of matching in the no-fault case in the nonlinear system depends on the assigned bandwidth of the robustness filter (in this case 2 rad/s). In Fig. 4, the effect of a slow drift fault on w is shown. Although both innovation elements are affected by this fault, they are sufficiently different to allow the fault to be detected quickly. The large transient in "d" is due to switching in the reliable sensor. Figure 4 shows the effect of a step offset fault again on the normal velocity signal w. In this case, the detection signal shows the nearly instantaneous correction of the fault. Figure 6 shows the effect of a scale factor variation. Again, the sensor fault onset and recovery are nearly instantaneous due to the large effect on the fault detect signal "d". Finally, in Fig. 7 the effect of a dead-band fault or "sticky" sensor is shown. With this fault the sensor gives a steady output until the difference between the actual measurement and the sensor output reaches some threshold, at which time the sensor output jumps to the new value. Note that in Figs. 5–7 the recovery from the fault occurs with such rapidity that no effect can be discerned on the measurement "b".

Conclusion

It has been shown that, by using the estimation error in the observation space, a powerful approach to fault diagnosis can be achieved. The new method constitutes a robust observer fault detection filter in which the effects of plant parameter variations and process disturbances are minimized. The robustness aspects have made it possible to place low thresholds for the rapid detection of faults. The use of the observation error subspace has removed the need to compute the state estimation vector, and hence designs can be implemented in real-time computer systems. The use of a bank of observers with different measurement pairs results in the additional advantage of enabling fault isolation. This has been demonstrated using a simulation example in which gyro faults are detected, isolated, and replaced within 200 ms. This recovery time scale is realistic and demonstrates that the method can be applied to a potentially unstable aircraft.

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